of the procedures. On the other hand, for users with a good background in calculus, linear algebra, and ordinary differential equations, the book makes it possible to become expert in numerical analysis by reading the text and doing the excellent problem sets that follow each major section. Solutions for most of the exercises, appear in Chapter 12. The authors present a wise balance of small-scale and large-scale computing methods. An instructor would have to make a judicious selection of the material to be covered by his class, since the book contains such a wealth of material treated with varying levels of mathematical sophistication.

This book is a beautifully written and improved translation of the authors' Swedish work published in 1969. The translator, Ned Anderson, is credited by the authors with improving the presentation. The chapter headings are:

Chapter 1	– Some General Principles of Numerical Calculation	Page 1
Chapter 2	– How to Obtain and Estimate Accuracy in Numerical Calculations	21
Chapter 3	– Numerical Uses of Series	60
Chapter 4	– Approximation of Functions	81
Chapter 5	– Numerical Linear Algebra	137
Chapter 6	– Nonlinear Equations	218
Chapter 7	– Finite Differences with Applications to Numerical Integration, Differentiation, and Interpolation	255
Chapter 8	– Differential Equations	330
Chapter 9	– Fourier Methods	405
Chapter 10	– Optimization	422
Chapter 11	– The Monte Carlo Method and Simulation	448
Chapter 12	– Solutions to Problems	465
Chapter 13	– Bibliography and Published Algorithms	536
	Appendix Tables	563
	Index	565

E. I.

19 [2.05].—G. G. LORENTZ, Editor, in cooperation with H. BERENS, E. W. CHENEY, L. L. SCHUMAKER, *Proceedings of an International Symposium Conducted by the University of Texas and the National Science Foundation, January* 22–24, 1973, Academic Press, Inc., New York, 1973, xiii + 525 pp., 24 cm. Price $17.00.

These proceedings contain six long articles and forty-nine shorter articles (eight typed pages maximum length). The shorter articles cover the entire field of approximation theory and include announcements of new results, summaries of previous work, and short research papers complete with proofs.

The long article by P. L. Butzer is a survey of the recent work by his colleagues in Aachen. There are eight books and 101 papers cited in the references, not counting works from outside Aachen. This seventy-page paper has nine distinct parts, the first four of which are the longest. These are

Basic approximation theory
Semigroup related results
Fourier analysis on R^n
Fourier analysis in Banach spaces
Approximation on compact manifolds

Best asymptotic constants
Kernels of finite oscillation
Spline approximation
Calculus for Walsh functions

The thirty-page paper by H. Berens and G. G. Lorentz considers Korovkin-type theorems for positive linear operators on Banach lattices. The paper primarily contains new results (extensions of previous results to Banach lattices). There are also many historical remarks, and thus the paper provides a thorough presentation of the results in this area.

The fifty-page paper by T. W. Gamelin outlines the ways in which the abstract theory of uniform algebras may be used to extend some classical approximation results to more general and/or abstract settings. The presentation revolves about three approximation problems: *Problem* I. Find conditions on a continuous complex-valued function $f(z)$ defined on K which guarantee that f can be approximated uniformly by functions analytic in a neighborhood of K. *Problem* II. What conditions on K guarantee that every continuous real-valued function on ∂K can be approximated uniformly by the real parts of functions which are analytic on a neighborhood of K. *Problem* III. Find conditions on a bounded analytic function $f(z)$ defined on K^0 which guarantee that f be a pointwise limit of a sequence of functions analytic in a neighborhood of K^0 and uniformly bounded on K^0.

The forty-eight-page paper by J. W. Jerome on multivariate approximation selects six topics from this field and summarizes the current state of approximation theory research for each of them. The six topics are

Generalized Peano kernel theorems
Spline approximations
Asymptotic estimates of widths
The finite element method
Rectangular grids in R^n
Convergence of Galerkin methods

The works of many people have been organized into a smooth and coordinated presentation.

The fourteen-page paper by D. J. Newman integrates the classical Jackson and Müntz theorems into a single setting. Consider approximation to continuous functions by $c_0 + \Sigma_{i=1}^n c_i x^{\lambda_i}$ where $0 < \lambda_1 < \lambda_2 < \cdots$. A typical result is that, if $\lambda_{i+1} - \lambda_i \geqslant 2$, then

$$\|f(x) - P_n(x)\| \leqslant K\omega_f[e^{-2\Sigma_{i=1}^n(1/\lambda_i)}],$$

where $P_n(x)$ is the best polynomial approximation and ω_f is the modulus of continuity.

The twenty-five-page paper by Daniel Wulbert considers the following problem: let $f(x)$ be continuous on the C^k-manifold M and assume $p(x) > 0$ on M. Is there a C^k function $g(x)$ defined on M so that $|f(x) - g(x)| < p(x)$ for all $x \in M$? The answer is "yes" if M is finite-dimensional and the extensions of this result to infinite-dimensional cases is the topic surveyed in this paper.

In summary, this is a valuable addition to the collection of books that a worker in approximation theory requires. The long (and some of the shorter) papers give an organized presentation of the current status of some important areas in approximation theory. The remaining papers give a broad cross section of the current activity in approximation theory which furnishes valuable insight into 'who considers what worthwhile and interesting'.

J. R. R.

20 [2.05.2].—R. P. FEINERMAN & D. J. NEWMAN, *Polynomial Approximation*, The Williams & Wilkins Co., Baltimore, Md., 1974, viii + 148 pp., 24 cm. Price $13.00.

A descriptive title for this book is "Degree of convergence for polynomial and rational approximation on the real line". This is a thorough and compact presentation of most of the known theory on this topic, the primary exclusions being those results that involve complex functions, analyticity, etc. There is a short (ten pages) chapter on the existence, uniqueness and characterization of best Tchebycheff approximations; and, otherwise, there is very little that does not relate directly to degree of convergence questions. Thus the scope of the book is rather narrow and it is not suitable as a general reference or text on approximation theory (even polynomial approximation).

As a special topics book, it is well done. The authors have organized the material well and concisely. There is a natural progression from traditional results to current research (to which one of the authors is a principal contributor) which the specialist in approximations theory will find readable and interesting. There are only thirty-eight items in the bibliography. The book is done economically as far as design, copy-editing and production are concerned; and only one misprint was noted (reference [25]).

J. R. R.

21 [2.05, 7].— HERBERT E. SALZER, *Laplace Transforms of Osculatory Interpolation Coefficients*, ozalid copy of handwritten ms. of six sheets, 11″ × 16″, deposited in the UMT file.

The Laplace transforms of the n-point $(2n - 1)$th-degree osculatory interpolation coefficients based on the integral points $i = 0(1)n - 1$, namely,

$$A_i^{(n)}(p) = \int_0^\infty e^{-pt}\{[L_i^{(n)}(t)]^2 [1 - 2L_i^{(n)\prime}(i)(t - i)]\}\, dt,$$

$$B_i^{(n)}(p) = \int_0^\infty e^{-pt}\{[L_i^{(n)}(t)]^2 (t - i)\}\, dt,$$

where

$$L_i^{(n)}(t) = \prod_{j=0, j\neq i}^{n-1} (t - j) \Big/ \prod_{j=0, j\neq i}^{n-1} (i - j),$$

are expressed exactly as functions of p, for $n = 2(1)9$. Both $A_i^{(n)}(p)$ and $B_i^{(n)}(p)$ underwent three functional checks that were made on the exact fractional coefficients of p^{-r}, $r = 1(1)2n$, on the final manuscript. All computations were performed with a desk calculator before 1962, except for the recent completion of the final checks by hand.

Given $f(i)$ and $f'(i)$, $i = 0(1)n - 1$, we have the approximation

$$\int_0^\infty e^{-pt} f(t)\, dt \approx \sum_{i=0}^{n-1} [A_i^{(n)}(p) f(i) + B_i^{(n)}(p) f'(i)].$$

AUTHOR'S SUMMARY

22 [2.25, 4, 7].—F. W. OLVER, *Asymptotics and Special Functions*, Academic Press, Inc., New York, 1974, xvi + 572 pp., 24 cm. Price $39.50.

This is a very satisfactory book, which combines sound mathematical analysis with